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ABSTRACT

Branched tubes have been employed to concentrate dilute solid-liquid suspensions. They utilize the nonuniform radial particle distribution resulting from lateral particle migration under laminar flow conditions to concentrate the suspension by directing the portion of the flow field near the main tube wall through a branch. Branched tube separation devices can be used in a continuous mode, and by employing a staged system, where each stage is made up of a set of branched tubes, it is possible to achieve high recovery, good concentration factors, and large throughput. Initial studies were directed toward obtaining large concentration factors, i.e., higher separations. This work examines the maximization of recovery, which is defined as the fraction of inlet particles recovered in the concentrated stream. Suspensions of *Spirulina platensis*, a microalgae, were concentrated using a single-branch system, and it was found that by using a 600-μm diameter main tube, the suspension could be concentrated 36 times using 7 stages with an overall recovery of 92.1%.

Key Words. Staged suspension concentration process; Inertial particle migration; Branched tube separators

INTRODUCTION

Branched tubes can be utilized to concentrate dilute solid-liquid suspensions. When a dilute suspension flows in the laminar regime through a tube,

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under certain conditions the suspended particles migrate laterally to an equilibrium radial position, resulting in a nonuniform distribution of particles across the radial cross-section of the tube. Branched tubes utilize the resultant concentration profiles to concentrate the suspension by directing the portion of the flow near the main tube wall through the branch. The phenomenon of lateral migration of suspended particles across the streamlines of flow is also referred to as inertial particle migration because it was demonstrated (2) that, if the inertial terms in the equations of motion of fluid are neglected, then no lateral (the direction perpendicular to the dominant flow direction) force can exist on a body of revolution in a nonuniform, unidirectional flow. The driving forces behind inertial migration originate from the specific pressure and shear stress distribution on the surface of a particle moving in a nonuniform flow field. When a dilute suspension of neutrally buoyant rigid spheres is transported along in Poiseuille flow (13), the particles migrate to an equilibrium radial region located approximately 60% of the distance from the tube axis to the tube wall. They called this phenomenon the "tubular pinch effect." The lateral forces are inertial lift due to the presence of shear, wall repulsion due to the lubrication effect, lift due to particle rotation, and, in the case of Poiseuille flow, a lift due to the added presence of velocity profile curvature (4). There is no applied magnetic or electric field, and the lift forces on the particles originate from purely hydrodynamical effects. The lift forces move the particle laterally (radially) until they reach an equilibrium radial position, where the net lift force on the particle is zero.

The phenomenon of inertial migration has also been observed for nonspherical particles. For a suspension of helical-shaped microalgae, *Spirulina platensis*, flowing through a 0.65-mm diameter tube (8), the particles migrated radially to a relatively narrow annular region. As the tube Reynolds number was increased within the laminar regime, the particles (microalgae) concentrated within an increasingly narrow region concentric with the tube axis. The primary parameters affecting lateral migration of particles in Poiseuille flow are particle to tube size ratio, the tube Reynolds number, the concentration of the suspension, the shape of the particle, the deformability of the particle, the density difference between the particle and fluid, and whether or not the particle is rotating.

Branched tube separators can be utilized to concentrate very dilute biological suspensions, which frequently occur in the bioprocessing industry (10). Tubes with branches were used as a separation device (11) for concentrating *Spirulina* microalgae suspensions. Concentration factors as high as 3 were obtained for laminar flows of *Spirulina* suspensions through branched tubes. The concentration factor is defined as the ratio of the exit particle concentration to that of the inlet concentration into the main tube of the separator. *Spirulina platensis* has a helical configuration and has a density close to that

of water. Its length may reach more than 1000 μm, and it has a helix diameter of approximately 25 μm. It may be used as a dietary supplement as it has a complete complement of essential amino acids. Among the existing techniques for the concentration of bioparticle suspensions, centrifugation and membrane filtration are often employed. An advantage of inertial migration-based separation in branched tubes is that the required pressure drop to obtain a specificed concentration factor is lower than that required by the membrane microfiltration technique (9). Membrane microfiltration also suffers from plugging of the membrane pores by the suspended particles, thereby decreasing the effective membrane surface area. Centrifugation is more expensive than filtration (1) and is often carried out in a batch mode.

Branched tube separation devices can be used in a continuous mode. By employing a staged system, where each stage is made up of a set of branched tubes, it may be possible to achieve high recovery, large concentration factors, and large throughputs. Efficient skimming in branched tubes is achieved when the fraction of flow through the branch is manipulated such that the location of the resultant dividing stream surface provides an optimum skimming of the flow field. The dividing stream surface is the boundary between the portion of the flow entering the branch and that which flows downstream in the main tube. A sketch of the orthogonal tube configuration used (11) to concentrate *Spirulina* microalgae suspensions is shown in Fig. 1. Q_i is the feed flow rate
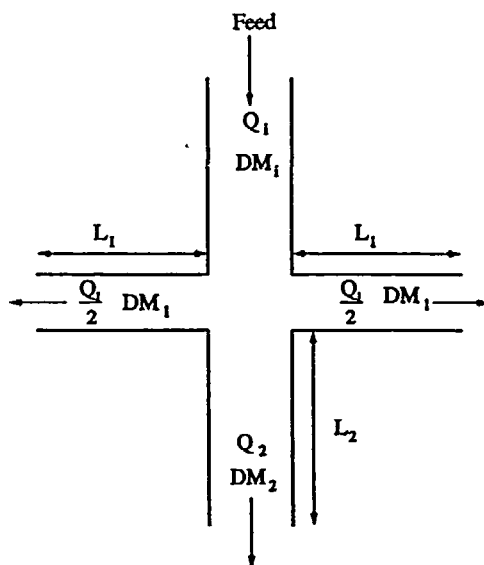

FIG. 1 Tube configuration and nomenclature.

to the tube, Q_1 is the total flow out of the two orthogonal branches, and Q_2 is the flow out of the main tube. Particle concentrations are expressed in terms of percent dry mass (% DM). One way to determine the optimum flow conditions and branch geometry for efficient skimming in each stage is to predict the concentration factors that may be produced for various flow conditions and for various branch geometries. The term "flow conditions" refers to the upstream tube Reynolds number and the fraction of the flow through the branch. The branch geometry includes the diameter of the main tube, the diameter ratio of the branch to the main tube, the junction shape, the ratio of the branch length to the downstream section length of the main tube, the number of branches, and the angle at which they intersect the main tube. In order to keep the construction of the branched tubes simple, there are no valves at the exits of the branches; and the ratio of the branch length to the downstream section length of the main tube is varied to provide the appropriate resistance for the desired flow splitting (10). This length ratio is referred to as the branch configuration.

In the development of a novel microalgae-based food (12) from *Spirulina* and agar (a marine polysaccharide), the starting mixture needs approximately 1% DM concentration of *Spirulina* suspension. The microalgal food is produced in sheet form. It is interesting to note that the inertial migration-based separation in branched tubes could be just the right harvesting step between the *Spirulina*-producing bioreactor and the sheet-making step. A design methodology was developed (6) for efficient stage wise concentration using branched tubes and applied to *Spirulina* suspensions. A design case study was conducted (6) for stagewise concentration of *Spirulina* suspensions from 0.01% DM (dry mass) to a final concentration of 1.0% DM, using an equidiametric branched tube system as shown in Fig. 1, with the diameter of the main tube being 650 μm. It was found in Ref. 6 that if the branch configuration in each stage is based on efficient flow skimming, then 13 stages are required to reach 1.0% DM concentration and an overall recovery of 29% is obtained; whereas for the case when the same branch configuration is used for all the stages, then the overall recovery is only 0.370%. Although relatively much higher recovery may be obtained for the variable branch configuration case, it is still not high enough from a commercial point of view. Also, in the design of latter stages of the stage train, the branch lengths required for efficient skimming were very long, which may make the branched tube construction very difficult. In this study we investigated the use of a slightly lower main tube diameter (600 μm) in order to obtain a tighter tubular pinch effect from inertial particle migration. Moreover, we employed a separator containing a single hole in the main tube for a branch, and varied the downstream length of the main tube to provide the appropriate flow ratios needed for efficient skimming in each stage. This would afford a simple construction

and assembly of multiple tubes in a staged process. All flows from the branched tube exit to atmospheric pressure in each stage. The concentrated suspension leaving the branched-tube system is collected and pumped to the next stage. Based on the Hagen–Poiseuille equation for pressure drop and the branched-tube system dimensions used in this work, a pressure drop of approximately 0.5 atm is expected for each stage. Also, using the methodology outlined in Ref. 6; an attempt is made to predict the concentration factors and recoveries using postulated upstream concentration profiles for *Spirulina* suspensions flowing in a tube.

METHODS

A suspension of *Spirulina platensis* microalgae was maintained in an air-cooled, continuously stirred, 6 L flask in standard Zerouk medium with $NaHCO_3$ as the main carbon source and at pH $\approx$ 9. Illumination was provided by two 4-ft long fluorescent bulbs mounted directly behind the flask. A Cole-Palmer masterflex peristaltic pump was used to flow the *Spirulina* suspension down through a vertical 600 μm diameter precision bore glass capillary tube. The branch consists of a circular 600 μm hole punched through a slit in the tube wall 20 cm downstream of an inlet (see Fig. 2). Collection of the side stream was facilitated by a glass jacket encasing the slit region, with an upper port open to the atmosphere and another port for collection. The exit streams were directly collected into graduated cylinders. The upstream tube Reynolds number, Re, is defined as $D\langle V\rangle/v$, where D is the diameter of the main tube, $\langle V\rangle$ is the average velocity in the upstream section of the main tube, and v is the kinematic viscosity of water. Modifications to the downstream length of the main tube were made with a water-cooled diamond tooth saw.

Suspension concentration was measured as percent dry mass of filaments and was determined by filtering a known weight of sample through a 1.0-μm Poretics polycarbonate membrane under vacuum. After filtering, the membranes were dried for 1 hour at 105°C and weighed with a Mettler AE160 analytical balance. The difference in the weight of the membrane before and after filtering and drying is taken as the weight of the dry filaments. The weight of the dry filaments as a percentage of the weight of the sample is then used as a measure of the suspension concentration and is referred to as % DM. Due to the time requirement for this concentration measurement method, a separate, although indirect technique was developed using the absorbance characteristics of the *Spirulina* suspension.

Using a Beckman DU-640 spectrophotometer, it was found that suspensions of *Spirulina* blanked with Zerouk medium have a high absorbance reading at a wavelength (λ) of 680 nm. A series of 30 *Spirulina* suspension samples with concentration ranging from close to 0% DM to 0.18% DM were prepared.
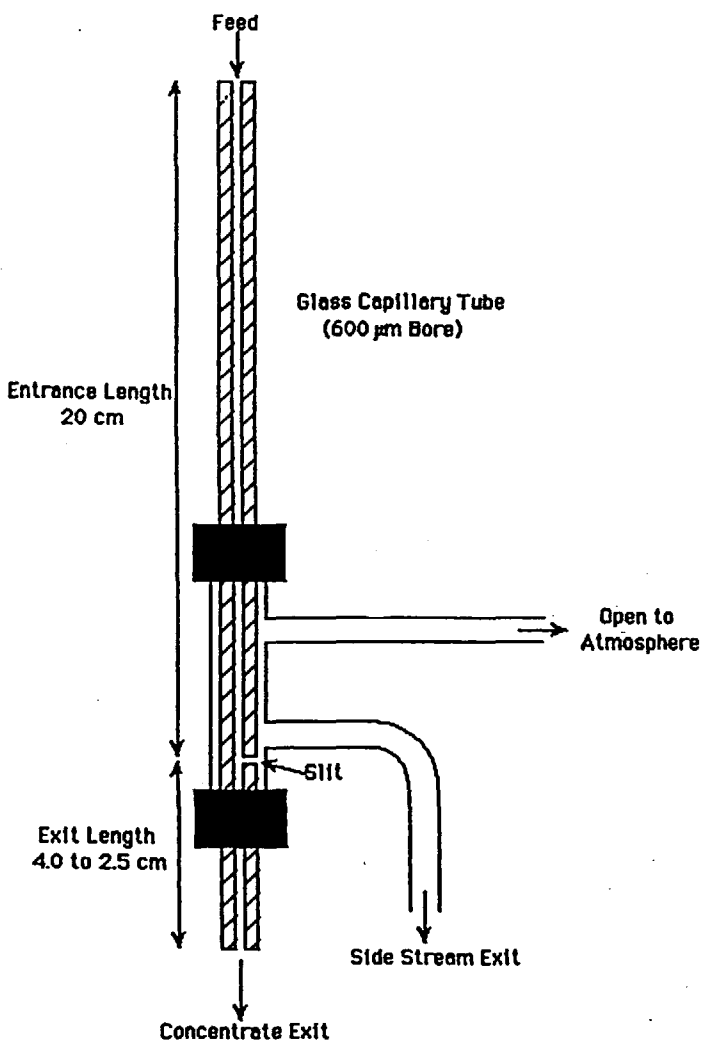
FIG. 2   The branched tube separator used in experiments.

The mean absorbance of each of these samples was determined by taking nine readings of each sample with agitation between each set of three readings to eliminate any effects due to settling. When the mean absorbance was plotted against % DM, a linear relationship between these quantities was observed (see Fig. 3). This linear relationship between the mean absorbance and %

DM was used as a fast and reliable means of determining the suspension concentration using the following correlation:

$$\% \text{ DM} = \frac{\sum_{i=1}^{n} A_i}{20.67n} \tag{1}$$

where n is the number of readings, A is the absorbance of the suspension, and the constant 20.67 is the slope of the linear relationship shown in Fig. 3. This relationship is valid for *Spirulina* concentrations up to 0.18% DM. For measuring concentrations beyond that, the sample may be diluted by a known factor so that its absorbance falls within the valid measurement range. However, in this work, for higher concentrations we directly determined the dry mass.

Each experimental run consists of pumping the suspension through the branched tube separator at various Re, and determining the flow rates and concentrations of streams exiting the branch and the downstream section of the main tube. *Spirulina* suspension concentrates in the downstream section of the main tube. The exit stream from the downstream section of the main tube is collected and then pumped through the modified separator in which the length of the downstream section of the main tube is reduced to provide efficient skimming. Each modification provides the subsequent stage in the stage train.
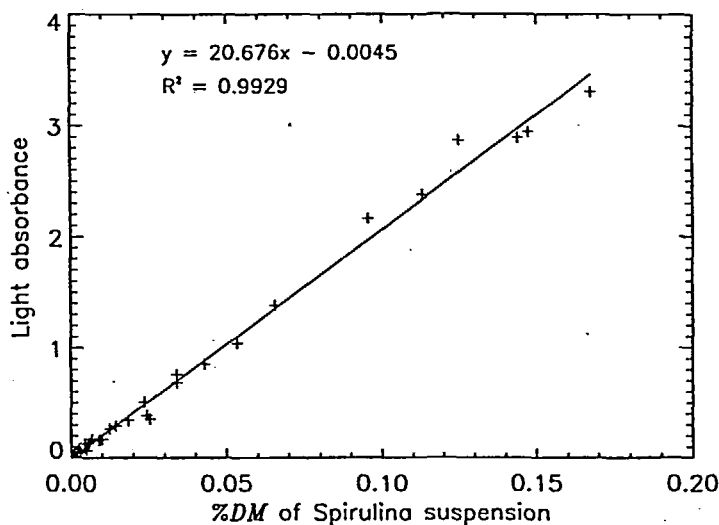

FIG. 3 Absorbance of light (680 nm) by suspensions of *Spirulina platensis* at various percent dry mass (% DM) concentrations.

RESULTS

Recovery is defined as the fraction of inlet *Spirulina* particles recovered in the stream exiting the main tube. In a commercial situation the dilute stream exiting the branch may be recycled back either to the reactor or to an appropriate earlier stage. The term "concentration factor" for *Spirulina* suspension refers to the ratio of the exit concentration from the main tube to the inlet concentration to the separator as *Spirulina* concentrates in the downstream section of the main tube. The experimental results for stagewise concentration of *Spirulina* suspensions in the branched tube separator shown in Fig. 2 is presented in Table 1. Refer to Fig. 1 for the nomenclature. Notice that in Stages 5 and 7 the downstream length of the main tube is reduced. This decreases the resistance in the downstream section of the main tube, resulting in lower flows through the branch hole, thereby reducing the flow ratio Q_1/Q_2. In the latter stages the feed concentration is significantly higher, resulting in the spreading of the particles toward the tube wall. Therefore, to obtain good recoveries in the latter stages, the flow ratio through the branch is reduced. Notice that in the last stage in Table 1, although Re $= 2505$, a laminar flow is expected because of the high suspension concentration, which will increase the effective suspension viscosity. The tube Re is defined using the viscosity of water in this work.

In order to determine the window of the operable flow ratio available for the branched tube system shown in Fig. 2, experiments were conducted in which water was pumped through the separator at various upstream tube Reynolds numbers, Re, and the flow ratios, Q_1/Q_2, were measured. The results are shown in Fig. 4. The factors contributing to the particular nature of variation shown in Fig. 4 include the unequal rate of change of resistance in the

TABLE 1
Experimental Results on Concentration of *Spirulina* Suspensions Using Branched Tubes

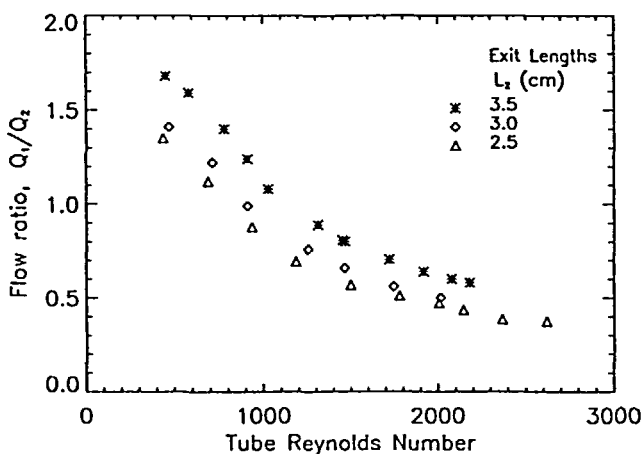| Stage number	Re	L_2 (cm)	Q_1/Q_2	% DM_i	% DM_1	% DM_2	Concentration factor	Stage % recovery
1	1445	4.0	0.86	0.015	0.0008	0.0289	1.89	100.0
2	1515	4.0	0.79	0.0289	0.0009	0.051	1.76	97.0
3	1557	4.0	0.77	0.051	0.0009	0.0875	1.72	99.9
4	1690	4.0	0.71	0.0875	0.0019	0.1472	1.68	96.0
5	1550	3.5	0.695	0.1472	0.0065	0.227	1.66	99.0
6	1750	3.5	0.59	0.227	0.005	0.373	1.64	100.0
7	2505	3.0	0.405	0.373	0.008	0.54	1.46	100.0
Overall % recovery								92.1

FIG. 4   The variation of flow ratio with tube Reynolds number for the single-hole branch tube system. Experiments were carried out for various exit lengths of the branched tube system shown in Fig. 2.

downstream of the main tube as compared to that in the branch hole, as the upstream flow rate changes, and the junction geometry. A general method for determining the frictional characteristics of such branched tube systems has been developed (6). The reason for using such high Re in the last stage (see Table 1) is to obtain lower flow ratio through the branch so as to maintain a high stage recovery. We were unable to reduce the exit branch length below 3 cm without significant sacrifice of recovery, probably because of end effects and/or gravity head changes. Therefore, we plan to carry out experiments to extend the stage train using a smaller branch hole diameter.

DISCUSSION

The overall recovery obtained in concentrating *Spirulina* suspension to 0.54% DM using the single-hole branch system is 92.1%, which is substantially higher than the predicted 36% in the case study (6). The reason for the substantially higher recovery is most likely the tighter tubular pinch effect that may have been obtained in the smaller diameter tube. By using the single-hole branch system, 7 stages are needed to reach 0.54% DM in contrast to approximately 10 stages needed for the two-branch system (6). It is interesting to note here that the flow ratios used for the single-hole branch system (see Table 1) are close to that recommended in the case study in Ref. 6. From our experience in using branched tube systems it is evident that every system

has unique frictional characteristics, which dictates the window of the operable flow ratio range available for the typical tube Reynolds number range (1200 $\leq$ Re $\leq$ 1800 for *Spirulina* suspensions) at which the tightest tubular pinch effect is observed. The unique frictional characteristics of a branched tube is the result of the unique junction geometry (roughness, curvature, etc.) obtained during its construction and the relative lengths of the branch and the downstream section of the main tube. It was fortuitous that the flow ratio window available for the single-hole branch system is close to that found in the two-branch system (6). We hope to develop general methods to determine the frictional characteristics of single-hole branch systems and other branched tube systems, which should eventually allow us to determine the branch configuration a priori.

In order to predict the concentration factors and recoveries in branched tube systems, the upstream concentration profile is required. The diameter of the main tube used in these experiments is slightly smaller than what was used in Ref. 6, for which the concentration profile was obtained using image analysis. The feed concentration of the suspension pumped through the tube was 0.03% DM, for which the concentration profiles were measured (6). In the present work, as the stage recoveries obtained for the smaller diameter tube are higher, it implies that fewer particles are present near the tube wall for the smaller diameter tube as compared to that in the 650-μm tube. Based on this observation, the upstream concentration profile available for the 650-μm diameter tube is modified so as to have fewer particles near the tube wall. The modification in concentration profile is done in such a way so as to preserve the total mass flow rates (see Fig. 5).

In order to determine the fraction of inlet particles leaving the exit of the main tube, the location of the dividing stream surface is needed. In the numerical computations of dividing stream surfaces (7) in two-branch systems, closed, circular dividing stream surfaces were found for Re = 150 and 250. These circular dividing stream surfaces were concentric with the main tube in two-branch systems. It was also shown that the circular dividing stream surfaces obtained on the basis of mass balance and the assumption of Poiseuille velocity profile at the tube junction is a very good approximation of the actual dividing stream surfaces at high Reynolds number in two-branch systems (7). For single-branch systems, closed dividing stream surfaces were observed for Re = 194 in the experimental observations of Ref. 3. At still higher Reynolds number, it is expected that these closed dividing stream surfaces in single-branch tubes will be nearly concentric with the main tube cross-section because of the large momentum of the core region of the flow. By integrating the upstream concentration profile with the flow velocity inside the circular dividing stream surface, the amount of material leaving the exit of the main tube can be determined. Using the methodology developed in
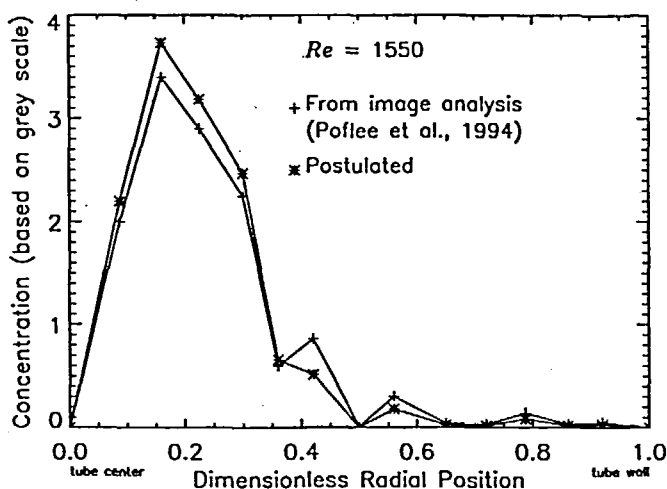
FIG. 5   The postulated upstream particle concentration profile for *Spirulina* suspensions flowing in a 600-μm diameter tube. The feed concentration of the suspension is 0.03% DM.

Ref. 7, the concentration factors and recoveries have been computed and are plotted in Fig. 6 as a function of the fraction of inlet flow through the branch. Notice that for those flow fractions for which large concentration factors are obtained, the recoveries are smaller (see Fig. 6b). Also plotted in Fig. 6 are the experimental observations presented in Table 1. It may be noticed that the experimental observations are close to that predicted using the upstream particle profiles for a feed concentration of 0.03% DM. However, the experimental observations plotted in Fig. 6 are for feed concentrations ranging from 0.015 to 0.373% DM. The close match between the experimental observations and the simulation in Fig. 6 implies that the qualitative nature of the upstream particle concentration profile of *Spirulina* suspension remains the same for feed concentrations up to 0.373% DM.

For still higher feed concentrations, the nature of the upstream concentration profile may be different from that shown in Fig. 5. At present we do not have the concentration profiles at high feed concentrations; however, it might be interesting to determine the potential for separation by branched tubes at the higher concentrations by postulating an upstream particle concentration profile. In Ref. 6, by using a rod model for *Spirulina* particles, it was determined that the maximum packing fraction in a tube of diameter much larger than a *Spirulina* particle is 0.092, which corresponds to 1.1% DM. Assuming that, due to inertial migration, *Spirulina* particles pack to a concentration of 1% DM at any particular radial location in the tube, a radial concentration
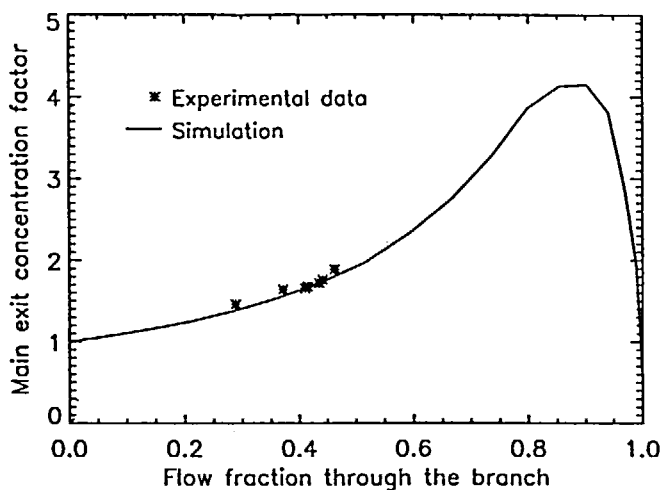
(a) Concentration factor
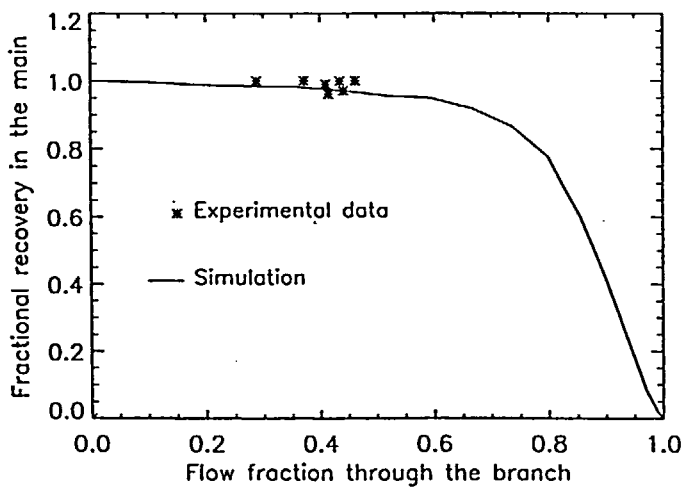
(b) Fractional recovery

FIG. 6   The predicted concentration factors and recoveries for *Spirulina* suspensions flowing in a 600-μm diameter tube.

profile may be postulated provided we know the equilibrium radial region of the tube occupied by the particles. It has been found (5) that spheres prevented from rotating occupy an equilibrium position of 0.14 tube radius from the tube axis. *Spirulina* has a helical configuration and it aligns in the direction of flow in the tube and does not rotate (9). Assuming that *Spirulina* particles are packed at a concentration of 1% DM in a radial region starting from 0.1 tube radius from the axis and extending toward the tube wall as the feed concentration increases, a concentration profile for the suspension at a given feed concentration may then be postulated. Also, at high suspension concentrations the velocity profile may deviate substantially from the parabolic velocity profile. By using the viscosity modeled developed in Ref. 6 for *Spirulina* suspensions, and the postulated upstream particle concentration profile, the velocity profile may be determined from the Navier–Stokes equations. Assuming steady-state conditions and that all the particles have reached their equilibrium radial position so that there is no radial and angular component of velocity, the momentum equations in the tube reduce to

$$\frac{1}{r}\frac{d(r\tau_{rz})}{dr} = -\frac{\Delta P}{L} \tag{2}$$

where r and z are the radial and axial directions respectively, τ is the shear stress, and $\Delta P/L$ is the pressure gradient in the tube. Then we substitute Newton's law of viscosity, $\tau_{rz} = -\mu dv_z/dr$ in Eq. (2), where v_z is the axial velocity and μ is the viscosity, which for *Spirulina* suspension will depend upon the particle concentration and thus may vary with the radial position in the tube. The Newtonian viscosity is then obtained from the viscosity model developed in Ref. 6 for *Spirulina* suspensions,

$$\mu = \mu_s(1 - \phi/\phi_m)^{-\eta\phi_m} \tag{3}$$

where ϕ is the volume fraction of particles in the suspension, ϕ_m is the maximum packing fraction, μ_s is the viscosity of the suspending fluid, and η is the intrinsic viscosity. For long *Spirulina* particles, $\phi = 0.092$, and $\eta = 13.76$ (6). Using Eqs. (2) and (3), and the postulated concentration profile, the velocity profile may be obtained. In Ref. 7 it was found that for tube Re ≥ 150, the dividing stream surfaces in branched tubes approach a closed, circular shape, which may be directly computed on the basis of mass balance in a branched tube system. Using circular dividing stream surfaces, and the velocity and concentration profiles, it is then possible to compute the concentration factors and the recoveries that may be obtained in branched tubes by using the procedure outlined in Ref. 6.

Figure 7 shows the postulated concentration profile and the computed velocity profile for a feed concentration of 0.54% DM, which corresponds to the exit concentration of Stage 7 of the experimental observations (see Table
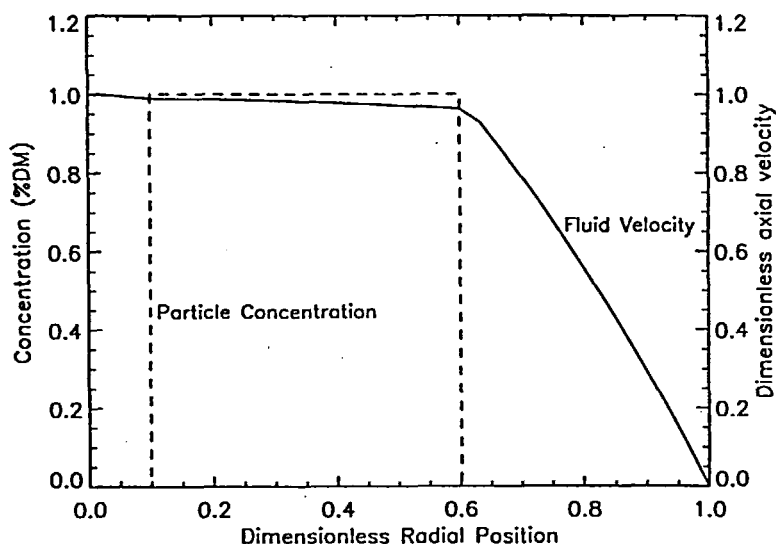
FIG. 7 The postulated upstream particle concentration profile for *Spirulina* suspensions and the corresponding velocity profile for Stage 8. The feed concentration of the suspension to the stage is 0.54% DM.

1). Figure 8 shows the predicted concentration factors and recoveries corresponding to the concentration and velocity profiles of Fig. 7. If we choose to operate Stage 8 at a flow fraction of 0.33 through the branch, so that 100% recovery is obtained, then the concentration factor for these operating conditions is 1.49. This results in an exit concentration of 0.807% DM from the main tube of Stage 8.

The postulated upstream concentration profile for the next stage (i.e., Stage 9) is shown in Fig. 9 along with the corresponding velocity profile. Using this information, the predicted concentration factors and recoveries at various flow fractions through the branch are plotted in Fig. 10. If we choose to operate at a flow fraction of 0.15, so as to get 100% stage recovery, then the exit concentration from the main tube will be 0.807 × 1.18 = 0.944% DM. On the other hand, if we choose to operate at a flow fraction of 0.25 so as to get an exit concentration of 1% DM, the stage recovery will drop to 93%. These computations show that there is a good potential to maintain stage recoveries close to 100% even at the higher concentrations, until approximately 1% DM final concentration is obtained.
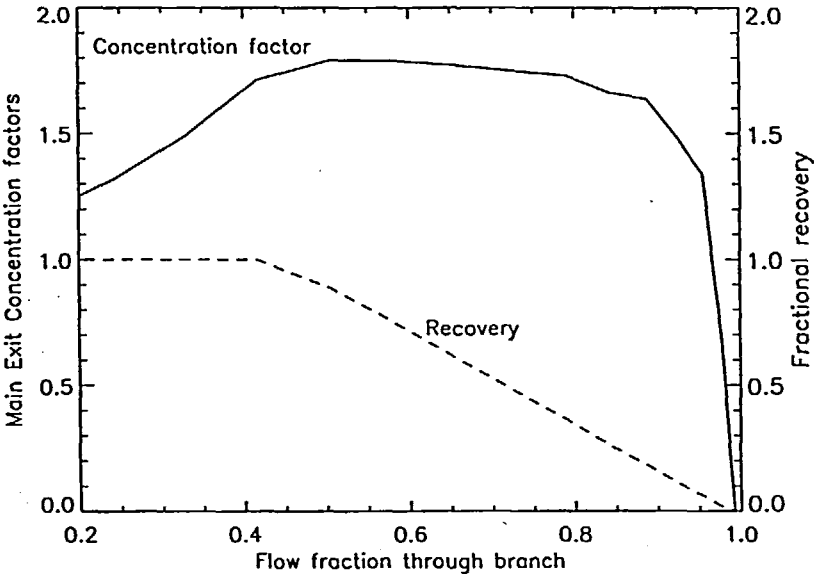
FIG. 8   The predicted concentration factors and recoveries for *Spirulina* suspensions for Stage 8.
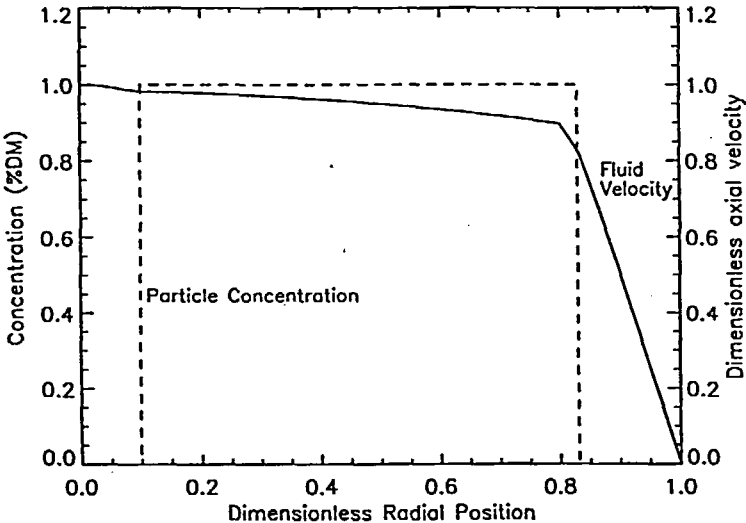

FIG. 9   The postulated upstream particle concentration profile for *Spirulina* suspensions and the corresponding velocity profile for stage 9. The feed concentration of the suspension to the stage is 0.8% DM.
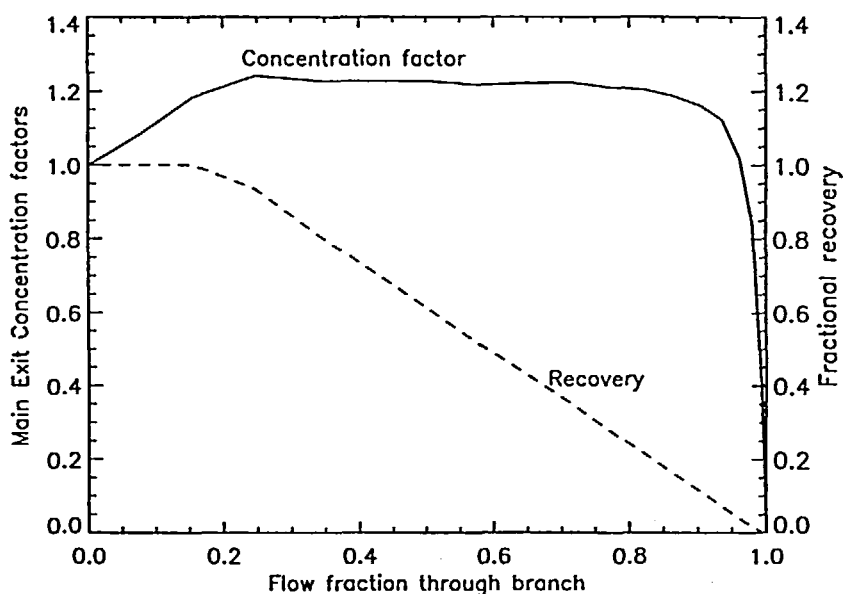
FIG. 10   The predicted concentration factors and recoveries for *Spirulina* suspensions for Stage 9.

CONCLUSIONS

By using a single-hole branch system with a main tube diameter of 600 μm, *Spirulina* suspension was concentrated 36 times (from 0.015 to 0.54% DM) and a large overall recovery (92.1%) was obtained. Moreover, it was found that only 7 stages are needed to achieve this, in contrast to the requirement of 10 stages predicted in the case study in Ref. 6.

Our analysis indicates that two more stages are needed to reach approximately 1% DM while still maintaining high recovery rates.

ACKNOWLEDGMENTS

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